3D Analytical Model for Estimation of Eddy Current Losses in the Magnets of IPM Machine Considering the Reaction Field of the Induced Eddy Currents

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Abstract—This paper proposes a closed-form 3D analytical model developed for the estimation of eddy current losses in an interior permanent magnet synchronous machine. Two different types of commonly employed interior permanent magnet machines are considered for the loss analysis. The results of the analytical model are compared with the results from 3D finite element analysis. The proposed model can be directly used to evaluate the eddy current magnet losses for various magnet segmentations in the axial and tangential directions. The influence of air-gap length, PWM carrier frequency, reaction field of induced eddy current and non-linearity of iron on the eddy current magnet losses have been studied in detail and the results are presented. The results from analytical and 3D finite element analyses were found to be in good concurrence with a maximum difference of 5.9 %.

Keywords—permanent magnets, eddy current loss, magnet segmentation, analytical model, finite element method, interior permanent magnet synchronous motor

I. INTRODUCTION

Interior permanent magnet (IPM) motors are widely used in high speed applications where the ratio of its nominal speed to maximum speed is in excess of 2 as in applications like hybrid and pure electric cars [1]. These motors are fed by a pulse-width modulated inverter which results in high frequency time harmonics. These harmonics cause eddy current losses in the magnet resulting in temperature rise and may result in irreversible demagnetization of the magnets. Hence, the calculation of the eddy current losses and its mitigation through segmentation can ensure the safety of magnets of PM machines at different loads and speeds.

Many previous publications have dealt with the calculations of eddy current losses in the magnets of PM machines, but mostly for surface mounted PM machines [2]-[7]. Magnetic field variations in the magnet are in the form of travelling waves in the surface mounted PM machines while they are pulsating wave form for IPM machines [8]. Therefore the analytical calculations of eddy current losses in the surface mounted PM machines cannot be directly applied to IPM machines. The eddy current losses calculation for the IPM machine is dealt in [9]-[13]. Analytical methods can help in reducing the computation time of eddy current calculations and in determining the appropriate magnet segmentation for magnet loss reduction. Analytical and numerical finite element analyses carried out in [9]-[13] didn’t show good concurrence. The air-gap length was neglected but is necessary for accurate estimation of eddy currents. In [14] the eddy current losses in IPM machines during flux weakening is evaluated using a loss coefficient. Eddy current losses in an axial flux PMSM has been dealt in [15] and [16]. In [8], a 2D analytical model developed by the authors which was extended to 3D using an end-effect factor. An analytical model in 3D for estimation of magnet eddy current losses in IPM machine is still missing.

In this paper, analytical and numerical finite element analyses of eddy currents in the magnets of IPM machines are presented. The analyses consider the effect of segmentation in the axial and tangential directions. The analytical model can be used in choosing the appropriate magnet segmentation numbers to reduce the eddy current losses in an IPM motor.

II. SPECIFICATIONS OF IPM MOTORS

Fig. 1 shows the two types of IPM motors considered for analysis [17]. Both motors have 8 poles and an internal and external stator diameter of 161.93 mm and 269 mm respectively. The air-gap length and axial length were 0.73 mm and 84 mm respectively. The magnet in both motors has a thickness of 6.5 mm while the width was 37.8 mm for Motor 1 and 18.9 mm for Motor 2.

The Cartesian co-ordinate system is chosen over the cylindrical co-ordinate system for the analysis as the magnets in an IPM machine are flat shaped, thereby reducing the complexity of the analysis. This assumption is valid if air-gap length is much smaller than the air-gap radius. In the present cases it holds true.

The following assumptions are made for analytical calculation of eddy currents in the magnets.

1) The relative permeability of iron is considered infinite ($\mu_r = \infty$).
2) Only the normal component of the flux density $B_y$ is considered ($B_x = B_z = 0$).
Only the components of current density along \( x \) and \( z \) axes, \( J_x \) and \( J_z \), are considered (\( J_y = 0 \)).

The effect of stator slotting is neglected.

The induced current densities \( J_x \) and \( J_z \) in the permanent magnet are shown in Fig. 2. The magnet width and depth are respectively represented by \( a \) and \( b \) in Fig. 2.

![Fig. 2: Induced current density in the magnets](image)

The simplified 2D cross-sections with induced currents in the magnet along the \( x \) and \( z \) directions are shown in Fig. 3 and 4.

![Fig. 3: Induced current density in the \( x \) direction](image)

![Fig. 4: Induced current density in the \( z \) direction](image)

For the case of segmented magnets, the appropriate magnet width \( a \) and depth \( b \) have to be considered. Let the numbers of segmented magnets in the circumferential and axial direction be represented by \( m \) and \( n \) respectively. If we consider the source field \( B_s \) in \( x \) and \( z \) directions according to Fig. 5 [18] [19],

\[
B_s = \frac{16}{\pi^2} B_{im} \sum_{n=1,3,5,...} \sum_{m=1,3,5,...} \frac{1}{n \cdot m} \cdot \sin \left( \frac{n \cdot \pi}{2} \right) \cdot \sin \left( \frac{m \cdot \pi}{2} \right) \cdot \cos \left( n \cdot \pi \cdot \frac{a}{2} \right) \cdot \cos \left( m \cdot \pi \cdot \frac{b}{2} \right)
\]

(4)

where \( B_{im} \) is the amplitude of \( B_s \).

Combining (2) and (3), the resulting differential equation for with \( B_s \) defined in (4) results as

\[
\frac{g_e}{d} \frac{d^2 B}{dx^2} + \frac{g_e}{d} \frac{d^2 B}{dz^2} - j \omega \mu_0 B_s = j \omega \mu_0 B_s
\]

(5)

Equation (5) is solved using Fourier series method. Therefore solution of \( B \) is given by
The induced eddy currents are:

\[ B = \frac{16}{\pi d} B_{sm} \cdot \sum_{n=1,3,5,7,...} \sum_{m=1,3,5,7,...} j \omega \mu_0 \sigma \cdot \sin \left( \frac{n \pi x}{2} \right) \cdot \sin \left( \frac{m \pi z}{2} \right) \cdot \cos \left( \frac{n \pi x}{2} \right) \cdot \cos \left( \frac{m \pi z}{2} \right) \cdot m \cdot \left( \frac{a}{d} \cdot \frac{m}{n} \right)^2 + \frac{a}{d} \cdot \left( \frac{m}{n} \right)^2 + j \omega \mu_0 \sigma \] (6)

The induced eddy currents are:

\[ \begin{align*}
J_s &= \frac{16}{\pi d} \frac{\theta_p}{d} B_{sm} \cdot \sum_{n=1,3,5,7,...} \sum_{m=1,3,5,7,...} j \omega \mu_0 \sigma \cdot \sin \left( \frac{n \pi x}{2} \right) \cdot \sin \left( \frac{m \pi z}{2} \right) \cdot m \cdot \left( \frac{a}{d} \cdot \frac{m}{n} \right)^2 + \frac{a}{d} \cdot \left( \frac{m}{n} \right)^2 + j \omega \mu_0 \sigma \\
J_s &= \frac{16}{\pi b} \frac{\theta_p}{d} B_{sm} \cdot \sum_{n=1,3,5,7,...} \sum_{m=1,3,5,7,...} j \omega \mu_0 \sigma \cdot \sin \left( \frac{n \pi x}{2} \right) \cdot \sin \left( \frac{m \pi z}{2} \right) \cdot m \cdot \left( \frac{a}{d} \cdot \frac{m}{n} \right)^2 + \frac{a}{d} \cdot \left( \frac{m}{n} \right)^2 + j \omega \mu_0 \sigma
\end{align*} \]

The eddy current loss in each segmented magnet can then be computed as:

\[ P = \frac{32}{\pi^2 d^2} \left( \frac{\theta_p}{d} \right)^2 \cdot \sum_{n=1,3,5,7,...} \left( \frac{a}{d} \cdot \left( \frac{m}{n} \right)^2 + \frac{a}{d} \cdot \left( \frac{m}{n} \right)^2 \right) \cdot \left( \frac{m}{n} \right)^2 + \left( \frac{a}{d} \cdot \left( \frac{m}{n} \right)^2 + j \omega \mu_0 \sigma \right)^2 \cdot \left( \frac{1}{(b \cdot n)^2} + \frac{1}{(a \cdot m)^2} \right) \]

The eddy current loss in a magnet segment of an IPM machine as given by the above is proportional to the volume of the magnet and its conductivity. It also varies as square of the flux density of the source field. The second term in the denominator of the loss equation, \( \omega \mu_0 \sigma \), represents the effect of reaction field due to the induced eddy current in the magnet.

**Estimation of \( B_{sm} \)**

Consider the simplified picture of the motor geometry as seen in Fig. 6. The value of \( B_{sm} \) is estimated using the scalar magnetic potential \([20]\). Let the MMF of the stator be aligned to the \( d \) axis. The instantaneous \( d \) axis MMF component \( V \) of a three phase winding can then be written as a function of circumferential angle \( \theta \) as

\[ V = V_m \cdot \sin(p \theta) \]

where \( p \) is the number of pole pairs and \( V_m \) is given by number of series turns/phase of the three phase winding \( N_p \), the maximum phase current in the winding \( I_{max} \) and the winding factor \( k_w \) as

\[ V_m = \frac{3}{\pi} \cdot k_w \cdot N_p \cdot I_{max} \]

The average \( d \) axis MMF across the magnet width of the stator winding can be calculated as:

\[ V_{av} = \frac{1}{p} \int \frac{\pi \cdot \theta_p}{2 \theta_p} V_m \cdot \sin(p \theta) \ d\theta \\
V_{av} = \frac{V_m}{p(\pi - 2 \theta_p)} 2 \cos(p \theta) \]

With the magnet and air-gap reluctances \( R_m \) and \( R_g \) defined as \( R_m = \mu_{o,ab} \) and \( R_g = \frac{2g}{\mu_{o,bg}(2g - 2d_p)} \), the scalar magnetic potential \( U_m \) across the magnet is evaluated as

\[ U_m = \frac{V_{av}}{R_m + R_g} \]

The flux density in the magnet \( B_{sm} \) can then be evaluated as

![Image](image-url)
The above equation gives a good estimate of $B_{sm}$. As the leakage fluxes are neglected, the true value of $B_{sm}$ is lower. It can be estimated accurately by a simple magneto-static 2D analysis. The resulting correction factor modifies the analytical value with the result from FE analysis.

III. RESULTS AND DISCUSSION

The validity of the analytical model is tested by comparing the loss results obtained with time harmonic/time-stepping 3D finite element method. As an example, on a fundamental time harmonic current with amplitude 250 A and frequency of 80 Hz at 1200 min⁻¹, a carrier harmonic with amplitude of 6.25 A (2.5 % of fundamental) at 10 kHz (due to PWM) on the rotor side is analyzed for losses in the magnets of the IPM motors. The stator and rotor lamination was considered with a relative permeability of 5000. An NdFeB magnet with a conductivity of 0.625 MS/m is considered for analysis. For the considered harmonic amplitude of 6.25 A, the flux density $B_{sm}$ was evaluated analytically at 15.3 mT. The estimated value of $B_{sm}$ by a simple 2D magneto-static analysis was found to be 14.7 mT for Motor 1 and 15 mT for Motor 2.

A. Effect of Segmentation

Fig. 7 and 8 show the comparison of magnet eddy current losses for various tangential and axial magnet segmentation numbers $m$, $n$ as obtained from analytical and FE method for Motor 1 and Motor 2. There is a good concurrence with a maximum difference of 5.9 %.

It is observed from Fig. 7 that for an unsegmented magnet in tangential direction ($m=1$) for Motor 1, an increase in the segmentation number along the axial direction for $1 < n < 9$ leads to increased eddy current losses in the magnets. Here only selection of $n > 10$ leads to reduction of eddy current losses in the magnets. The region $1 < n < 5$ is characterized as so-called inductance-limited region where the eddy current losses increase with increasing segmentation number [6], [8]. However, the region $6 < n < 12$ is characterized as resistance-limited region where the eddy current losses decrease with increasing segmentation number. The strength of the reaction field from the magnet in the inductance limited region is higher compared to the resistance limited region resulting in distortion of the source field. It is also observed from Fig. 7 that for $m = 2$ and $m = 3$ increasing $n$ reduces the eddy current losses in the magnets. Here the inductance-limited region is absent and only the resistance-limited region exists.

Fig. 8 shows that for Motor 2 only a resistance-limited region exists and increasing segmentation number $n$ results in reduced eddy current losses. It is observed from Fig. 8 and 9 that as far as magnet eddy current losses are concerned, Motor 2 is version of Motor 1 with $m = 2$, provided the segmented magnet dimensions are maintained in proportion.

Fig. 9 and 10 show the 3D plot of eddy current losses with different segmentation numbers $m$ and $n$. The appropriate selection of the segmentation number is finally decided by the motor designer taking into account manufacturing costs and efficiency specification.

Fig. 11 and 12 show the induced eddy current density plot as obtained by 3D FE analysis with $m=n=2$ for Motor 1 and $m=n=1$ for Motor 2, respectively. The skin effect can be clearly noticed with the eddy currents concentrated at the magnet edges.

Fig. 13 and 14 show the induced eddy current density plot along the $x$ and $z$ directions with $m=1$ and various $n$ for Motor 1 as obtained by the analytical model. It can be inferred from the above figures that with increasing segmentation number $n$ up to 5, the induced current densities $J_x$ and $J_z$ are nearly constant while the area through which the current flows is increased. This leads to increased losses in the magnet as it was noticed in Fig. 7.

Fig. 15 shows the plot of the variation of the peak flux density in the magnet with $B_{sm}$ of 14.7 mT for $m=n=1$ with Motor 1 as obtained by the analytical model. The skin effect can be clearly noticed as the flux density reduces inside the magnet along the magnet width and depth.
Fig. 9: 3D plot of magnet losses variation with segmentation, Motor 1

Fig. 10: 3D plot of magnet losses variation with segmentation, Motor 2

Fig. 11: Induced current density plot for $m = n = 2$, FEA, Motor 1

Fig. 12: Induced current density plot for $m = n = 1$, FEA, Motor 2

Fig. 13: Induced current density along the x direction, Motor 1

Fig. 14: Induced current density along the z direction, Motor 1
B. Influence of air-gap length

The influence of air-gap length for the calculation of induced eddy current losses in the magnets of IPM machine was dealt in another paper on 2D analytical loss model by the authors [8]. The influence of air-gap length is studied by replacing the effective air-gap length \( g_e = g + d \) by \( g_e = d \) in the loss equation. The effect on losses with and without the consideration of the air-gap length is seen in Fig. 16 for Motor 1. Neglecting the air-gap length leads to lower eddy current losses as the reaction field effect from the magnet is under-estimated. This holds true for Motor 2.

C. Influence of reaction field

The influence of reaction field from the induced eddy currents in the magnets on the losses was dealt in the above mentioned paper. Neglecting the effect of reaction field leads to under-estimation of magnet losses especially in the inductance-limited region. In the resistance-limited region, with higher \( n \), the difference is minimal as the effect of reaction field is reduced significantly. Fig. 17 shows the effect on losses with and without the consideration of the reaction field on the evaluated total eddy current loss for Motor 1 and Motor 2 with \( m=1 \) and \( n \) varied from 1 to 12.

D. Influence of magnet conductivity

The conductivity of the magnet has a very dominant effect on the eddy current induced in the magnets. The comparison of magnet losses with NdFeB and SmCo (with a conductivity twice of NdFeB) magnets for with segmentation \( m=1 \) and \( n \) varied from 1 to 21 is shown in Fig. 18 for Motor 1 and Motor 2. It is observed with increased conductivity the loss curve shifts to the right.

E. Influence of iron non-linearity

The analytical model assumed the relative iron permeability as infinite and hence it was possible to obtain a closed-form expression for the eddy current losses. 3D FE analysis has been used to calculate the losses taking the non-linearity into consideration. For stator and rotor, lamination
material M330-35A was used [21]. Fig 19 shows the variation of losses with the consideration of non-linear effects of iron lamination for Motor 1. Since a nonlinear 3D time stepping model is very time consuming, the model is done for \( n = 6 \) and \( m \) varied for 1 to 6 with only 1/6th of the model simulated thereby reducing the simulation effort.

It has to be noted that at the operating condition of 250A fundamental peak, the RMS current loading corresponds to 1500 A/cm. So at the chosen operating point, the motor is in a highly saturated condition. The eddy current losses with saturation lies in the range of \( \pm 11\% \) when compared to the linear case depending on the tangential segmentation number \( m \). It is clear from the analysis that analytical model can be still employed as the good approximation to estimate the eddy current losses in the magnet of IPM even with saturation.

**F. Influence of carrier frequency**

The region corresponding to inductance- or resistance-limited is determined by the frequency of the carrier harmonic with all the other parameters remaining constant. Fig. 20 and 21 show the variation of the loss for different carrier frequencies for Motor 1 and Motor 2 respectively. With increasing carrier frequency the loss curve shifts to the top and right.

Fig. 22 and 23 show the induced current densities \( J_x \) and \( J_z \) in the magnet at different carrier frequencies for Motor 1 with \( m = n = 1 \). The skin effect at high frequencies can be clearly observed with current concentrated at the magnet edges and the magnitude of current density increasing with increasing frequency. At frequencies below 5 kHz the behavior of induced current density \( J_x \) can be approximated to be linear along the magnet width.
G. Loss estimation with inverter excitation

The proposed analytical loss model estimates the losses in the magnets of IPM machine for given amplitude of harmonic current. With inverter excitation several currents at different frequencies result in the stator. The loss with individual harmonic current needs to be estimated and the total magnet loss is then the sum of individual loss components.

IV. CONCLUSIONS

This paper proposed a 3D analytical model for quick estimation of eddy current losses in the magnets of IPM motors taking into account the reaction field of the induced eddy currents. The model analyzed for two different types of commonly used IPM machines taking into account the effect of circumferential and axial magnet segmentation. The analysis showed that an unskilled choice of the segmentation number can actually lead even to increased magnet losses in the machine resulting in higher magnet temperature and thereby resulting in reduced performance with the risk of irreversible demagnetization. It was observed that the reaction field, if neglected, results in big errors in the estimation of magnet losses especially in the inductance-limited region. The effect of air-gap length has a significant effect on the magnet losses. The actual loss was higher with the air-gap length considered. The effect of carrier frequency on the magnet loss was also analyzed. Increasing the carrier frequency means increased losses in the magnets. Hence the choice of carrier frequency can be a major factor in deciding the eddy current losses in the magnets. Having proved the validity of the proposed analytical model at different switching frequencies, the analysis can be extended to evaluate the magnet eddy current losses with inverter excitation.

ACKNOWLEDGMENT

This work is funded by the Federal Ministry for Economic Affairs and Energy (BMWi) within the ATEM program. The authors are responsible for the contents of this publication.

REFERENCES


